Exam in VO "Statistical Physics", WS 2024/25

12.12.2024

1. $\Omega(E)$ is the number of microstates for a given energy E.

   a. Express the fundamental assumption of statistical physics through $\Omega(E)$.

   b. Discuss why we request that E has to jitter by a small amount δE.

   c. How is the entropy related to $\Omega(E)$?

2. Consider $\Omega_1(E_1)$, $\Omega_2(E_2)$ for two isolated systems. Answer the following questions:

   a. What happens if the two systems are brought into thermal contact?

   b. Write down the expression $\Omega_{\text{tot}}(E_{\text{tot}})$ for the combined system.

   c. What is the condition for thermal equilibrium?

   d. Discuss why the entropy for the total system increases.

   e. How can the expression for $\Omega_{\text{tot}}(E_{\text{tot}})$ be simplified for sufficiently large systems where fluctuations can be neglected?

   f. Argue why $\frac{\partial S}{\partial E}$ is related to the inverse temperature and not the temperature itself.

3. Consider an entropy of the form $S(E) = Nf(E)$. What is an *extensive* quantity? Compute the first and second derivatives of $S(E)$ and $\Omega(E)$. Why is it advantageous to use an expansion of the entropy for large values of N?

4. Consider the partition function $Z(E, V)$. How is it defined? Give the relations between Z and:

   a. the mean energy,

   b. the energy fluctuations,

   c. the entropy.

   d. How is the free energy F defined? How is it related to the canonical partition function? When should one use F rather than the energy E?

5. Consider two systems with $\Omega_1(E_1, V_1, N_1)$ and $\Omega_2(E_2, V_2, N_2)$ that can also exchange particles. What is the chemical potential μ, how is it related to the entropy S? Derive the condition for thermal equilibrium between the two systems.

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6. The grand-canonical ensemble $\Phi = F - \mu N$ has the total differential

$$d\Phi = -S\,dT - p\,dV - N\,d\mu\,.$$

   a. On which quantities does Φ depend?

   b. Show how to obtain S and p from Φ.

   c. Φ can be written as the product between an extensive quantity (which one?) and an intensive function. On which quantities does the intensive function depend?

7. Compute the partition function for a photon gas. Show the steps needed for the calculation in full length. Discuss for a single mode the limits of low and high temperature (with respect to what?).

8. Derive the grand-canonical partition function for non-relativistic electrons in three dimensions. What is the Fermi energy? Obtain the general expression from which the chemical potential $\mu(T)$ can be derived. Discuss why the heat capacity C_V at low temperatures is so small in comparison to a classical gas.