Exam in UE "Statistical Physics", WS 2024/25

04.12.2024

Exercise 1. (10 Points) The equation of state for a classical ideal gas has the form

$$S = k_B N \log\left(\frac{V}{N}\right) + N f(T)\,. \tag{I}$$

Here N is the number of particles, V is the volume, and $f(T)$ is a function that only depends on the temperature T. Two volumes V_1 and V_2 are separated by a wall and are filled by ideal gases of different particle species, see Eq. (I) for the respective equations of state, with the same temperature T and pressure p. When removing the walls, the system comes to equilibrium.

a. Explain why the entropy of the combined system increases when removing the wall. Discuss why the term $Nf(T)$ can be neglected when computing the entropy change from Eq. (I).

b. Compute the entropy change ΔS and show that $\Delta S > 0$.

c. Repeat the analysis for gases of the *same* particle species, i.e., initially both volumes V_1 and V_2 are filled by gases consisting of the same particle species. Show through explicit calculation that $\Delta S = 0$.

Exercise 2. (10 Points) A particle with energy $E = p^2/(2m) + kx^4$ is in contact with a heat bath at temperature T. Use the equipartition theorem

$$\left\langle p\frac{\partial E}{\partial p}\right\rangle = \left\langle x\frac{\partial E}{\partial x}\right\rangle = k_B T$$

$\langle \dots \rangle = \int d^3p \int d^3x \; e^{-\beta E} \dots$

to compute the mean energy $U = \langle E \rangle$.

Exercise 3. (10 Points) Consider a system of free, non-interacting fermions of spin $1/2$ in two dimensions.

a. Compute the corresponding density of states $g(E)$.

b. Compute the Fermi energy for a given number of fermions N inside an area A.

c. Obtain the relation between the particle area density N/A in terms of the fugacity $z = e^{\beta\mu}$ and the thermal de-Broglie wavelength

$$\lambda = \sqrt{\frac{2\pi\hbar^2}{mk_BT}}\,.$$

You don't have to determine the fugacity explicitly.