

You are allowed to use paper and pencil.

Problem 1: (12 Points) Describe the following concepts in a concise way (focus on the most important things, no more than a couple sentences are needed)

- Bisection method
- rectangular rule
- iterative solution of a system of linear equations
- The order of a method for solving ODEs
- Aliasing in frequency analysis
- Cholesky decomposition

Problem 2: (4 Points) Show that the relative condition number of subtraction is infinity.

Problem 3: (8 Points) Describe the secant and the Newton-Raphson method for the solution of a non-linear equation.

Problem 4: (6 Points) When calculating a numerical derivative using the central difference formula, what is the best stepsize that we can employ and why?

Problem 5: (4 Points) Suppose you have some datapoints x_i, y_i with $i = 1, \dots, 5$. What's the difference between a polynomial fit and a polynomial interpolation?

Problem 6: (6 Points) You have a matrix A and an approximate eigenvector \mathbf{v} . How can you calculate the corresponding eigenvalue? How can you improve the accuracy of \mathbf{v} (such that it's closer to an exact eigenvector)?

Problem 7: (6 Points) Describe the Golden ratio search for finding extrema of functions of one variable.

Problem 8: (6 Points) Describe the Gradient descent method for finding minima of functions. Does it have a relation to the Newton-Raphson method for root search? When does it find the global minima of a function?

Problem 9: (4 Points) You need to calculate an integral $\int_a^b f(x)dx$ where $f(x)$ is some smooth function, and you are interested in the result to high accuracy. What method would you use and why? What method would you use, if you have $f(x)$ calculated from the numerical solution of a differential equation?

Problem 10: (6 Points) You have two datasets, a_i and b_i with $i = 1, \dots, N$. You need to calculate their convolution,

$$c_i = \sum_{k=1}^N a_k b_{i-k},$$

where you consider the datapoints to be periodic: $b_k = b_{N+k}$. How can you calculate c_i for $i = 1, \dots, N$ and what is the cost of the calculation?