KFU Graz
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**Computational Physics** WS 24-25

Exam
You are allowed to use paper and pencil.

**Problem 1: ( 10 Points)** Describe the following concenpts in a concise way (focus on the most important things, no more than a couple sentences are needed)

- Simpson rule
- rounding errors
- Nyquist frequency
- Givens rotation
- gradient descent

**Problem 2: ( 6 Points)** What are the convergence rates of the Bisection and the Newton-Rhapson method? Give also the proof for your statements.

**Problem 3: ( 6 Points)** Can you give a fixed point iteration for the solution of the equation

$$\frac{x}{2} + x^2 = \sin x? \tag{1}$$

(we are interested in the smallest positive root around 0.5). Does the method converge?

**Problem 4: ( 4 Points)** Show that the trapezoidal integration rule is exact for linear functions, and that it is not exact for quadratic polynomials.

**Problem 5: ( 6 Points)** When calculating a numerical derivative using the forward difference formula, what is the best stepsize that we can employ and why?

**Problem 6: ( 6 Points)** You have a large positive definite matrix and you are interested in its two largest eigenvalues. How can you calculate them? Does the method also work for a non positive definite matrix?

**Problem 7: ( 6 Points)** Define the LU decomposition of a matrix. What is pivoting and why might it be needed?

**Problem 8: ( 4 Points)** Why is the iterative solution advantageous for the solution of linear systems defined in terms of a sparse matrix? (or is it?)

**Problem 9: ( 6 Points)** Show that using the Forward Euler method to solve the initial value problem $\dot{y} = f(y,t),\ \ y(0) = a$ with the stepsize $\Delta t \ll 1$, one should expect that we get $y(1)$ up to an error of $O(\Delta t)$.

**Problem 10: ( 8 Points)** You are throwing a ball of some mass $m$ and radius $r$ from zero initial height with an initial velocity vector $\mathbf{v} = (v_0, 0, v_0)$. There air resistance is given by a force $F_r = -\eta \mathbf{v} v$. (There is of course also gravity). What should be $v_0$ such that the ball lands in distance $d$ (on a horizontal plane)?
Describe the steps of a numerical solution to this question.