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Functional Analysis 2023/2024

Example 1. (5 Points)
Let l^∞ be the vector space of all bounded sequences. Moreover, let

$$\|\cdot\| : l^\infty \to \mathbb{R}$$
$$(x_n)_{n\in\mathbb{N}} \mapsto \sum_{n=1}^{10} |x_n|.$$

Is this mapping a norm? Check all four requirements of a norm!

Example 2. (5 Points)
Consider the vector space of polynomials in the interval $[-1,1]$ and the linearly independent system of monomials $\{1, x, x^2, ...\}$. Use the Gram-Schmidt orthonormalization procedure to determine the first two orthonormal polynomials with respect to the scalar product (Gegenbauer polynomials):

$$(f,g) = \int_{-1}^{1} dx(1-x^2)f(x)g(x).$$

Example 3. (5 Points)
Let $f(x) = |\sin(x)|$ for $x \in [-\pi, \pi]$.

a) (4 Points) Determine the real Fourier series of the function f.
b) (1 Point) Does the Fourier series converge uniformly? Explain your answer!

Example 4. (5 Points)
Calculate the Fourier transform of the function

$$f(x) = \begin{cases} \cos(x) & \text{if } x \in [-\pi, \pi] \\ 0 & \text{else.} \end{cases}$$

Useful formulas: For $\alpha, \beta \in \mathbb{R}$ holds

- $2\sin(\alpha)\cos(\beta) = \sin(\alpha+\beta) + \sin(\alpha-\beta)$ ex 3
- and $\cos(\alpha) = \frac{e^{i\alpha}+e^{-i\alpha}}{2}$. ex 4