1 Example (5 Points)

Let l^2 be the vector space of all summable sequences, i.e.

$$l^1 = \left\{ (x_n)_{n \in N} : \sum_{n=1}^{\infty} |x_n| < \infty \right\}.$$

Moreover, let

$$\begin{aligned} \|\cdot\| &: l^1 \to \mathbb{R} \\ (x_n)_{n\in N} &\mapsto \sum_{n=4}^{\infty} |x_n|. \end{aligned}$$

Is this mapping a norm? Check all four requirements of a norm!

2 Example (5 Points)

Consider the vector space of polynomials in the interval $[-1, 1]$ and the lincarly independent system of monomials $\left\{1.x, x^2, \ldots\right\}$. Use the Gram-Schmidt orthonormalization procedure to determine the first two orthonormal polynomials with respect to the scalar product (Gegenbauer polynomials):

$$(f, g) = \int_{-1}^{1} dx \left(1 - x^2\right)^{1/2} f(x)g(x)$$

Hint: Make use of trigonometric substitutions of the form $x = \sin u$ to solve the integrals.

3 Example (5 Points)

Let $f(x) = |x|$ for $x \in [-\pi, \pi]$.

1. (4 Points) Determine the real Fourier series of the function f.
2. (1 Point) Does the Fourier series converge uniformly? Explain your answer!

4 Example (5 Points)

Calculate the Fourier transform of the function

$$f(x) = \begin{cases} \sin(x) & \text{if } x \in |-\pi, \pi| \\ 0 & \text{else.} \end{cases}$$

Useful formulas: For $\alpha, \beta \in \mathbb{R}$ holds

1. $2\sin(\alpha)\cos(\beta) = \sin(\alpha + \beta) + \sin(\alpha - \beta)$
2. $2\cos(\alpha)\cos(\beta) = \cos(\alpha + \beta) + \cos(\alpha - \beta)$
3. and $\sin(\alpha) = \frac{e^{20} - e^{-+\alpha}}{2i}$

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