Your name

Title of your thesis

BACHELOR'S THESIS

Bachelor's degree programme:
Mathematics

Supervisor

DDr. rer. nat. MMag. Max Mustermann

Institute for Mathematics
University of Technology Graz

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1 Introduction

Definition 1.1 (Continuos)**.** Let (X, d_X), (Y, d_Y) be metric spaces, $f\colon X \to Y$ a map and $x_0 \in X$. We say f is *continuous* in x_0, if

$$\forall \varepsilon > 0 : \exists \delta > 0 : \forall x \in X : d_X(x, x_0) < \delta \implies d_Y(f(x), f(x_0)) < \varepsilon.$$

Proposition 1.2 (Sequence criterion for continuity)**.** *Let (X, d_X), (Y, d_Y) be metric spaces, $f\colon X \to Y$ a map and $x_0 \in X$. Then f is continuous in x_0 if and only if, for every sequence $(x_n)_{n\in\mathbb{N}}$ in X which converges to x, the sequence $(f(x_n))_{n\in\mathbb{N}}$ converges to $f(x)$.*

Proof. A classical proof via a double implication, where each implication is done by proving the contrapositive. □

Definition 1.3 (Limit of a function)**.** Let (X, d_X), (Y, d_Y) be metric spaces, $\Omega \subseteq X$, $f\colon \Omega \to Y$ a map, $x_0 \in X$ an accumulation point of Ω and $y \in Y$. We say f has the *limit* y in x_0, if

$$\forall \varepsilon > 0 : \exists \delta > 0 : \forall x \in (B_\delta(x_0) \cap \Omega) \setminus \{x_0\} : d(f(x), y) < \varepsilon.$$

In this case we write $\lim_{x\to x_0} f(x) = y$.

Theorem 1.4 (Limit criterion for continuity)**.** *Let (X, d_X), (Y, d_Y) be metric spaces, $\Omega \subseteq X$, $f\colon \Omega \to Y$ a map and $x_0 \in \Omega$ an accumulation point of Ω. Then f is continuous in x_0 if and only if*

$$\lim_{x\to x_0} f(x) = f(x_0).$$